

Chapter 10 / **Example 19****Finding roots of a complex number**

Use your calculator to find all the solutions of the equation $z^5 = 2 + 5i$ in Cartesian form.

Open a new document and add a Calculator page.

Find the fifth root of $2 + 5i$ and store it as z .

Type $\sqrt[5]{2+5i}$, then press $\boxed{\text{ctrl}}$ $\boxed{\text{var}}$ ($\boxed{\text{sto} \rightarrow}$) z and press $\boxed{\text{enter}}$.

To enter i press $\boxed{\pi}$ and select i from the menu.

The calculator gives just one solution of the multiple valued root.

Calculator screen showing the expression $\sqrt[5]{2+5i} \rightarrow z$ and the result $1.36087+0.330227i$.

Where $z = r \operatorname{cis} \theta$, $\sqrt[n]{z} = \sqrt[n]{r} \operatorname{cis} \frac{\theta + 2k\pi}{n}$, $k = 0, 1, 2, \dots, n-1$.

Find r and store it by typing $|z|$, press $\boxed{\text{ctrl}}$ $\boxed{\text{var}}$ ($\boxed{\text{sto} \rightarrow}$) r and press $\boxed{\text{enter}}$..

Enter the modulus function by pressing $\boxed{| \cdot |}$ and selecting $\boxed{| \cdot |}$ with the trackpad.

Calculator screen showing the expression $|z| \rightarrow r$ and the result 1.40036 .

Find θ and store it by typing $\boxed{\text{menu}}$ 2: Number | 9: Complex Number Tools | 4: Polar Angle, type z , close the parentheses, press $\boxed{\text{ctrl}}$ $\boxed{\text{var}}$ ($\boxed{\text{sto} \rightarrow}$) θ and press $\boxed{\text{enter}}$.

To enter θ press $\boxed{\pi}$ and select θ from the menu.

Calculator screen showing the expression $\text{angle}(z) \rightarrow \theta$ and the result 0.238058 .

Now use the formula for arguments to find all the solutions and convert these to Cartesian form.

Type $r \times e^{\left(\theta + \frac{2\pi}{5}\right)i}$ and press $\boxed{\text{enter}}$.

Since the default format of complex numbers is Cartesian, the GDC converts from polar to Cartesian format without needing to enter any additional command.

Calculator screen showing the expression $r \cdot e^{\left(\theta + \frac{2\pi}{5}\right)i}$ and the result $0.106466+1.39631i$.

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To repeat this calculation, select it and press **enter**.

Edit the expression, changing 2 to 4 and press **enter**.

$ z \rightarrow r$	1.40030
$\text{angle}(z) \rightarrow \theta$	0.238058
$r \cdot e^{i \left(\theta + \frac{2 \cdot \pi}{5} \right)}$	0.106466 + 1.39631 · i
$r \cdot e^{i \left(\theta + \frac{4 \cdot \pi}{5} \right)}$	-1.29507 + 0.532738 · i

Repeat changing 4 to 6 and press **enter**.

$r \cdot e^{i \left(\theta + \frac{2 \cdot \pi}{5} \right)}$	0.106466 + 1.39631 · i
$r \cdot e^{i \left(\theta + \frac{4 \cdot \pi}{5} \right)}$	-1.29507 + 0.532738 · i
$r \cdot e^{i \left(\theta + \frac{6 \cdot \pi}{5} \right)}$	-0.906862 - 1.06706 · i

Repeat changing 6 to 8 and press **enter**.

$$z_1 = 1.36 + 0.330i, z_2 = 0.106 + 1.40i, z_3 = -1.30 + 0.533i,$$

$$z_4 = -0.907 - 1.07i, z_5 = 0.735 - 1.19i.$$

$r \cdot e^{i \left(\theta + \frac{4 \cdot \pi}{5} \right)}$	-1.29507 + 0.532738 · i
$r \cdot e^{i \left(\theta + \frac{6 \cdot \pi}{5} \right)}$	-0.906862 - 1.06706 · i
$r \cdot e^{i \left(\theta + \frac{8 \cdot \pi}{5} \right)}$	0.734596 - 1.19222 · i